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## A SIMPLE ADAPTIVE NULLING TECHNIQUE USING PHASE-ONLY CONTROL

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### A Simple Adaptive Nulling Technique Using Phase-Only Control

#### 1. INTRODUCTION

The increase in the number density and power of jammers has stimulated much interest in adaptive antenna pattern control. Some techniques employ an adaptive control at each array element. This is practical for communication array antennas, which typically have few elements. On the other hand, large phased-array radar antennas have far too many elements to be economically feasible to have one control loop per element. Antennas with extremely low sidelobes, less than 45 dB, below the peak of the mainbeam have some protection against jammers because of their low sidelobes, but the cost to achieve these low sidelobes is high. In addition, the increase in beam width associated with a low sidelobe taper has the undesirable effect of increasing the resulting clutter.

An attractive alternative approach to reducing the sensitivity to jammer interference is to employ the beam-steering phase shifters inherent to electronically scanned phased-array antennas as the null weighting elements. The following technique not only takes advantage of the beam-steering phase shifters, but also produces a null with only five power measurements. An initial measurement is made to establish a reference, then four additional power measurements are

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made with a cancellation beam formed in the desired null direction. Those beams have the same amplitude and the phase is varied in 90° increments. The perturbations to the phase shifters are determined from some simple calculations, which produce a null in the desired direction.

#### 2. FORMULATION OF ANALYTIC CANCELLATION BEAMS

Analytic cancellation beams are a convenient means to form adaptive nulls in the far-field pattern of large phased-array antennas. Beams constrain the number of control variables to be proportional to the number of desired null locations with one amplitude and phase (of the cancellation beam) per null. Also, this approach forces the perturbations in the far-field to occur only in prescribed directions, thus maintaining the integrity of low-sidelobe antenna patterns.

An analytic beam is formed and controlled by adjusting the excitation of each array element in a specified manner. A physical beam, on the other hand, is formed by hardware such as the Butler matrix or Rotman lens and is controlled by adjusting the excitation at a single beam port. The formulation of analytic cancellation beams that are formed and controlled by adjusting only the beam-steering phase shifters follows.

Consider the linear array shown in Figure 1 consisting of N equispaced isotropic elements. The far-field pattern, p(u), for this array is

$$p(u) = \sum_{n=1}^{N} W_n \exp\{j d_n u\}$$

where

 $W_n$  = element excitation,  $u = kd \sin \theta$ ,  $k = 2\pi/\lambda$ , with  $\lambda$  being the wavelength, d = element spacing, and  $d_n = n - (N + 1)/2$ .

In general, the element excitation is complex with an amplitude taper  $a_n$  and phase  $Q_n = -d_n u_s$  to steer the beam in the direction  $u = u_s$ , resulting in unperturbed or quiescent element excitations

$$W_n = W_{qn} = a_n \exp \left[-j d_n u_s\right]$$
.

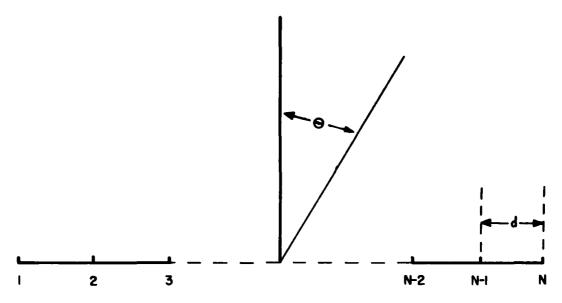


Figure 1. Array Geometry

Suppose we now perturb just the element phase by  $\Delta\phi_{\,n}.$  The perturbed coefficients become

$$W_n = W_{qn} \exp \left[ j \Delta \phi_n \right]$$

Furthermore, if we require the perturbed pattern to have nulls in M specified directions, the  $\Delta\,\phi_n$  satisfy the nonlinear equations

$$p(u_m) = \sum_{n=1}^{N} W_{qn} \exp [j(d_n u_m + \Delta \phi_n)] = 0$$
 ,  $m = 1, 2, ... M$  .

If the phase perturbations are small,  $\exp[j\Delta\phi_n] \approx 1 + j\Delta\phi_n$ . With this requirement, the nonlinear equations reduce to the linear equations

$$p(u_m) = \sum_{n=1}^{N} W_{qn} (1 + j\Delta \phi_n) \exp[jd_n u_m] = 0, \quad m = 1, 2, ... M$$
 (1)

that consist of the quiescent pattern,  $p_q(u_m)$  plus perturbations. Therefore,

$$p(u_m) = p_q(u_m) + j \sum_{n=1}^{N} \Delta \phi_n \exp[j d_n u_m] = 0$$
, m = 1, 2, ... M , (2)

where

$$p_{q}(u_{m}) = \sum_{n=1}^{N} W_{qn} \exp \{j d_{n} u_{m}\}$$
.

By a proper choice of the  $\Delta\phi_n$ , we can concentrate the pattern perturbations in the directions of the desired nulls, thus forming a cancellation pattern in those directions. In a large array, the number of array elements, N, typically exceeds the number of desired nulls, M. Thus further constraints of the  $\Delta\phi_n$  are required to obtain a unique solution. Shore and Steyskal derived a number of solutions for an ideal array with a purely real pattern. One solution is of the form

$$\Delta \phi_n = a_n^{-1} \sum_{m=1}^{M} b_m \sin [d_n (u_s - u_m)]$$
 (3)

The pattern perturbation that results for this case consists of M sinc beams with amplitude  $b_{\rm m}/2$  and steered in the  $u=u_{\rm m}$  direction for m=1,2... M, plus an additional set of sinc beams also with amplitude  $b_{\rm m}/2$ , but steered in the  $u=-u_{\rm m}+2u_{\rm s}$  directions. The far-field pattern for this solution upon applying Euler's identity for the sine function is

$$p(u) = p_{q}(u) + \sum_{m=1}^{M} (b_{m}/2) \left\{ \sum_{n=1}^{N} \exp \left[ j d_{n} (u - u_{m}) \right] - \sum_{n=1}^{N} \exp \left[ j d_{n} (u + u_{m} - 2u_{s}) \right] \right\}.$$
(4)

The above solution for an ideal array is extended in Reference 2 to form cancellation beams with complex coefficients,  $b_m$  exp [j $\beta_m$ ]. If we let

$$\Delta \phi_n = a_n^{-1} \sum_{m=1}^{M} b_m \sin [d_n (u_s - u_m) + \beta_m]$$
 (5)

<sup>1.</sup> Shore, R. A., and Steyskal, H. (1982) Nulling in Linear Array Patterns With Minimization of Weight Perturbations, RADC-TR-82-32, AD A118695.

O'Brien, M. J. (1984) Phase-Only Adaptive Nulling With Cancellation Beams for Large Arrays, RADC-TR-84-154, AD

the far-field pattern,

$$p(u) = \sum_{n=1}^{N} W_{qn} (1 + j \Delta \phi_n) \exp [j d_n u]$$
.

becomes

$$p(u) = p_{q}(u)$$

$$+ \sum_{m=1}^{M} (b_{m}/2) \left\{ \sum_{n=1}^{N} W_{qn} a_{n}^{-1} \exp \left[ d_{n} (u - u_{m} + u_{s}) + \beta_{n} \right] - \sum_{n=1}^{N} W_{qn} a_{n}^{-1} \exp \left[ d_{n} (u + u_{m} - u_{s}) - \beta_{n} \right] \right\}$$

$$= p_{q}(u) + \sum_{m=1}^{M} b_{m} p_{m}(u) . \qquad (6)$$

Upon substituting the Wan, the first summation on n simplifies to

$$(b_{m}/2) \exp [j\beta_{m}] \sum_{n=1}^{N} \exp [jd_{n}(u-u_{m})]$$
,

which represents a beam with a complex coefficient  $b_n \exp [j\beta_m]$  that is steered in the  $u = u_m$  direction and the second summation on n represents a concomitant beam with a complex coefficient  $-b_m \exp [-j\beta_m]$  and is steered in the  $u = -u_m + 2u_s$  direction.

The nulling requirement is now reduced to determining the  $\mathbf{b}_m$  and  $\boldsymbol{\beta}_m$  , such that

$$p(u_{\ell}) = p_{q}(u_{\ell}) + \sum_{m=1}^{M} b_{m} p_{m}(u_{\ell}) = 0, \quad \ell = 1, 2, ... M.$$

Thus, in order to produce a null in a given direction, the sum of the cancellation beams must cancel the quiescent pattern in that direction. In other words, the sidelobes of the other beams combine with the peak of the beam steered in the desired direction to produce the null.

A method for reducing the interaction between the cancellation beams is presented in Reference 3. Briefly, this approach is to form a new cancellation beam in the  $u_l$  direction for  $l = 1, 2, \ldots M$ , consisting of a weighted sum of the  $p_m(u)$  described above. This composite beam is

$$B_{\ell}(u) = \sum_{m=1}^{M} a_{m\ell} \sin \left[ d_{n} (u_{s} - u_{m}) + \beta_{\ell} \right] \exp \left[ j d_{n} (u - u_{s}) \right] . \tag{7}$$

To reduce the beam interaction we require that

$$B_{\ell}(u_{m})=0 \quad , \quad m \neq \ell$$

with  $a_{\ell\ell} = 1$  and  $\beta_{\ell} = 0$ .

Once these two forms of the cancellation beams are specified, what remains is to determine the beam coefficients  $b_m$  and  $\beta_m$  adaptively to form nulls in the perturbed pattern.

#### 3. MEASUREMENT TECHNIQUE

This method of adaptively determining the beam coefficients uses a series of received power measurements, followed by a calculation on these measurements to find  $b_m$  and  $\beta_m$ . An interference signal received by an antenna at a given direction is weighted by the sidelobe level of the antenna in that direction. The aim of this measurement technique is to find the phase and amplitude of the sidelobe in the direction of the interference signal, based on measurements that include the strength of the interference signals as well as the desired signal.

To formulate the expressions for the beam coefficients in terms of the measured - wers we first consider the simple case of one interference source. Let the total power received via the quiescent pattern be represented by A with

$$A = S |p_q(u_s)|^2 + J_1 |p_q(u_1)|^2$$

<sup>3.</sup> O'Brien, M. J. (1984) Nulling in Array Patterns With Orthogonal Analytic Cancellation Beams, RADC-TR-84-38, AD A142719.

#### where

S = intensity of the desired signal,

u\_ = direction of the desired signal,

J, = intensity of the interference signal, and

u, = direction of the interference signal.

To compute the magnitude and phase of the quiescent pattern in the direction of the interference by using measurements of the total received power, we form an analytic beam in the direction  $u = u_1$ . We chose the initial amplitude,  $b_I$ , of this beam so that its peak is equal to the expected (rms) sidelobe level of the quiescent pattern.

Therefore,

$$b_{I} = 10^{(SLL/20)}/|p_{1}(u_{1})|$$

where

SLL = rms sidelobe level.

We proceed by taking power measurements with cancellation beam phases of 0°, 90°, 180°, and 270°. If the received power for each  $\beta$  is  $m_{\beta}$ , then

$$m_{\beta} = S |p_q(u_s) + b_I p_1(u_s)|^2 + J_1 |p_q(u_1) + b_I p_1(u_1)|^2$$

For a low sidelobe pattern  $p_q(u_s) >> b_{IP_1}(u_s)$ . Consequently, the measured power consists of a nearly constant residual power due to the desired signal plus the power due to the interference. Therefore,

$$m_{\beta} = R_1 + J_1 |p_{\alpha}(u_1) + b_1 p_1 (u_1)|^2$$
,

where the residual power  $R_1$  is

$$R_1 \approx S |p_q(u_s)|^2$$

The jammer's contribution to the received voltage at each measurement is shown in Figure 2. The interference is weighted by the sum of the quiescent side-lobe level  $p_q(u_1) = A_1 \exp \left[ j \theta_1 \right]$  and the cancellation beam with amplitude  $B_1 = |b_1 p_1(u_1)|$  and phases  $\beta_m = 0^\circ$ , 90°, 180°, and 270°. The received power with these values of  $\beta_m$  is

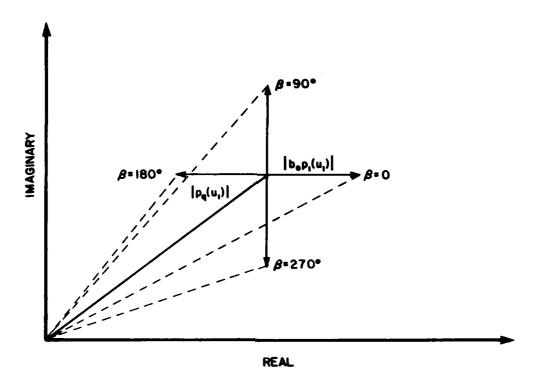


Figure 2. Received Interference Intensity Due to Cancellation Beam Phase Settings

$$m_0 = R_1 + J_1 (A_1^2 + B_1^2 + 2A_1 B_1 \cos \theta_1)$$
 (8a)

$$m_{180} = R_1 + J_1 (A_1^2 + B_1^2 - 2 A_1 B_1 \cos \theta_1)$$
 (8b)

$$m_{90} = R_1 + J_1 (A_1^2 + B_1^2 + 2 A_1 B_1 \sin \theta_1)$$
 (8c)

$$m_{270} = R_1 + J_1 (A_1^2 + B_1^2 - 2A_1B_1 \sin \theta_1)$$
 (8d)

By subtracting Eq. (8b) from Eq. (8a) and Eq. (8d) from Eq. (8c) we obtain

$$m_0 - m_{180} = 4 J_1 A_1 B_1 \cos \theta_1$$
 (9a)

and

$$m_{90} - m_{270} = 4 J_1 A_1 B_1 \sin \theta_1$$
 (9b)

from which we obtain

$$\theta_1 = \tan^{-1} [(m_{90} - m_{270})/(m_0 - m_{180})]$$

Since the phase of the cancellation beam is desired to be opposite to that of the quiescent pattern in the direction of interference,  $\beta_1 = \theta_1 \pm 180^{\circ}$ .

With the cancellation beam phase determined we must now extract the amplitude, which means we must first determine b<sub>1</sub>, as follows. The sum of the four measurements gives

$$m_0 + m_{90} + m_{180} + m_{270} = 4[R_1 + J_1(A_1^2 + B_1^2)]$$
 (10)

Squaring and then adding Eq. (9a) and Eq. (9b) gives

$$(m_0 - m_{180})^2 + (m_{90} - m_{270})^2 = J_1^2 (4 A_1 B_1)^2$$
 (11)

Also note that

$$R_1 = A - J_1 A_1^2 . (12)$$

These three equations can then be solved for A<sub>1</sub>, the amplitude of the quiescent pattern in the direction of the interference given by

$$A_{1} = \frac{\left[ (m_{0} - m_{180})^{2} + (m_{90} - m_{270})^{2} \right]^{1/2}}{\left[ m_{0} + m_{90} + m_{180} + m_{270} - 4 A \right]} B_{1} \qquad (13)$$

In order for the cancellation beam to form a null in the specified direction, the peak of the cancellation beam must equal the amplitude of the quiescent pattern in that direction. Consequently the desired cancellation beam amplitude coefficient becomes

$$b_1 = A_1/p_1(u_1)$$

We also know that

$$B_1 = b_1 p_1 (u_1)$$
.

Therefore, the desired amplitude coefficient is

$$b_1 = A_1 b_I / B_1 \quad ,$$

which from Eq. (13) becomes

$$b_1 = \frac{b_1 \left[ (m_0 - m_{180})^2 + (m_{90} - m_{270})^2 \right]^{1/2}}{\left[ m_0 + m_{90} + m_{180} + m_{270} - 4A \right]} . \tag{14}$$

We have shown above that both the amplitude and phase for a single cancellation beam can be obtained in terms of the received power for five measurements, with an initial estimate for the cancellation beam amplitude coefficient. The above technique is now extended to produce multiple nulls in the far-field pattern. Here the total power received via the quiescent pattern becomes

$$A = S |p_q(u_s)|^2 + \sum_{m=1}^{M} J_m |p_q(u_m)|^2$$

for M interference signals with

and

Following the same procedure described above for a single null, we form an analytic beam in the direction  $u = u_m$ , take power measurements for  $0^{\circ}$ ,  $90^{\circ}$ ,  $180^{\circ}$ , and  $270^{\circ}$ , and then compute the proper phase and amplitude to produce the null. This process is accomplished sequentially for  $m = 1, 2, \ldots M$ . Thus, M nulls are obtained with 5M measurements.

Here, the power received through the pattern perturbed by the cancellation beam formed in the u = u, direction is given by

$$m_{\beta} = S |p_{q}(u_{s}) + b_{I}p_{I}(u_{s})|^{2} + \sum_{m=1}^{M} J_{m} |p_{q}(u_{m}) + b_{I}p_{I}(u_{m})|^{2}$$

We can express this measured power as the sum of constant residual power plus the received power due to the  $\ell^{th}$  interference providing

$$b_{I} p_{\ell} (u_{s}) < < p_{q} (u_{s})$$

and

$$\mathbf{b}_{\mathbf{I}} \mathbf{p}_{\mathbf{I}} (\mathbf{u}_{\mathbf{m}}) << \mathbf{p}_{\mathbf{q}} (\mathbf{u}_{\mathbf{m}})$$
 ,  $\mathbf{m} \neq \mathbf{I}$ 

As described earlier for the single null case, for low sidelobe patterns the first condition is satisfied and for the present we will assume the second is also satisfied. Thus,

$$m_{\beta} \approx R_{\ell} + J_{\ell} |p_{q}(u_{\ell}) + b_{l} p_{\ell}(u_{\ell})|^{2}$$

where the residual power R, is independent of the cancellation beam.

If we let  $p_q(u_l) = A_l \exp [j \theta_l]$  and  $B_l = |b_l p_l(u_l)|$ , then the received power corresponding to the various  $\beta_m$  is given by

$$\begin{split} \mathbf{m}_{0} &= \mathbf{R}_{l} + \mathbf{J}_{l} \left\{ \mathbf{A}_{l}^{2} + \mathbf{B}_{l}^{2} + 2 \, \mathbf{A}_{l} \, \mathbf{B}_{l} \cos \theta_{l} \right\} \\ \mathbf{m}_{180} &= \mathbf{R}_{l} + \mathbf{J}_{l} \, \left\{ \mathbf{A}_{l}^{2} + \mathbf{B}_{l}^{2} - 2 \, \mathbf{A}_{l} \, \mathbf{B}_{l} \cos \theta_{l} \right\} \\ \mathbf{m}_{90} &= \mathbf{R}_{l} + \mathbf{J}_{l} \, \left\{ \mathbf{A}_{l}^{2} + \mathbf{B}_{l}^{2} + 2 \, \mathbf{A}_{l} \, \mathbf{B}_{l} \sin \theta_{l} \right\} \\ \mathbf{m}_{270} &= \mathbf{R}_{l} + \mathbf{J}_{l} \, \left\{ \mathbf{A}_{l}^{2} + \mathbf{B}_{l}^{2} - 2 \, \mathbf{A}_{l} \, \mathbf{B}_{l} \sin \theta_{l} \right\} \end{split} .$$

Again, following the approach for the single null we see that

$$\beta_{\ell} = \tan^{-1} \left[ (m_{90} - m_{270})/(m_0 - m_{180}) \right] \pm 180^{\circ}$$

and

$$b_{\ell} = b_{I} \frac{\left[ (m_{0} - m_{180})^{2} + (m_{90} - m_{270})^{2} \right]^{1/2}}{\left[ m_{0} + m_{90} + m_{180} + m_{270} - 4 A \right]}$$

Once the phase shifters are set using these beam coefficients, the power is again measured to establish the initial received power level, A, for the succeeding nulling operation.

The results of this technique are sensitive to the accuracy of the assumption that the residual power remains constant during the measurements with the four quadrature beam phases.

Let us address first the assumption,  $b_I p_I(u_g) \ll p_q(u_g)$ . Recall that the peak of the cancellation beam occurs at  $u = u_I$  and has a value  $|b_I p_I(u_I)|$  equal to the rms sidelobe level of the quiescent pattern. Consider a low sidelobe pattern with an rms sidelobe level -40 dB with respect to the mainbeam. Also recall the cancellation beam approximates a sinc beam with -13-dB peak sidelobe levels. In the worst case this peak sidelobe would occur in the direction of the desired signal, and  $b_I p_I(u_g)$  would be 53 dB below the peak of the main beam.

The second assumption,  $b_I p_I (u_m) << p_q (u_m)$ ,  $m \ne I$ , can be satisfied by nulling with the composite cancellation beams  $B_I (u)$  described in Section 2 rather than the individual sinc beams  $p_I (u)$ . Recall these composite beams are formulated such that  $B_I (u_m) = 0$ , m = I.

#### 4. SIMULATION RESULTS

To demonstrate the quality of null achievable with the measurement technique, we applied it to a numerical model of an experimental phased-array antenna. The computer simulation allows us to extract information about the nulling performance that is not obtainable in an experimental model. For example, to compute the total received power, we must compute the power received from each source separately, while in the experiment we measure only the total power. Consequently, we can test the validity of the assumption that the residual power is relatively constant during each nulling operation.

The array being modeled consists of 80 H-Plane sectorial horn elements spaced at 0.518-wavelength intervals. The antenna uses 8-bit phase shifters to electronically steer the beam. Figure 3 shows the measured amplitude distribution. We used these measured values to determine the phase shifter settings according to Eq. (5) and to compute the radiation patterns.

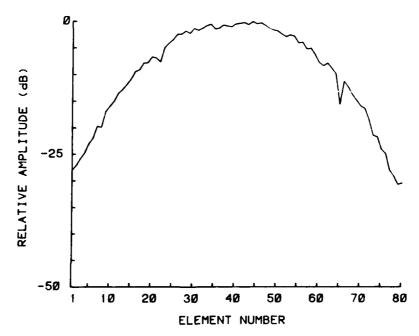


Figure 3. Measured Amplitude Distribution of the 80-Element Linear Array Being Modeled

The sets of results presented here are intended to demonstrate the effectiveness of the technique with the presence of certain errors encountered in a physical system.

In the first example, we considered a phase error free antenna with analog phase shifters. Figure 4 shows the cancellation pattern, as computed using Eq. (6), along with the quiescent pattern. The desired null location is at 15° from broadside. Notice the phase perturbations concentrate energy in the  $\theta$  = -15° as well as in the desired direction. Recall from Eq. (6) that the phase perturbations form two beams; one steered in the desired direction and another concomitant beam steered to u = -u<sub>m</sub> + 2u<sub>s</sub>. The perturbed pattern is compared with the quiescent pattern in Figure 5. The perturbed pattern has not changed perceptibly except in the null and concomitant beam directions. The nulling process suppressed the -37-dB sidelobe by an additional 28 dB resulting in a null 67 dB below the mainbeam. Figure 6 displays the relative jammer power vs the number of iterations where one iteration corresponds to one power measurement. The phase of the quiescent pattern at  $\theta$  =15° was -14.0° and the cancellation beam phase computed from the measurement technique  $\beta$  = 166.38°, which differs only slightly from the ideal phase of 166°.

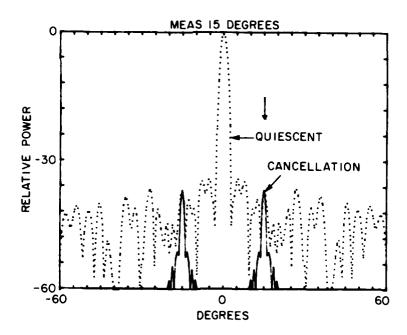


Figure 4. Cancellation Pattern and Quiescent Pattern for a Null Placed at  $\theta$  = 15°

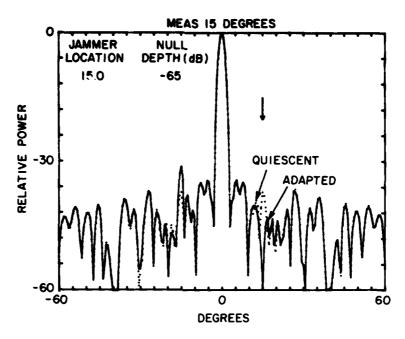


Figure 5. Adapted Pattern Compared to Quiescent Pattern for a Null at  $\theta$  = 15°

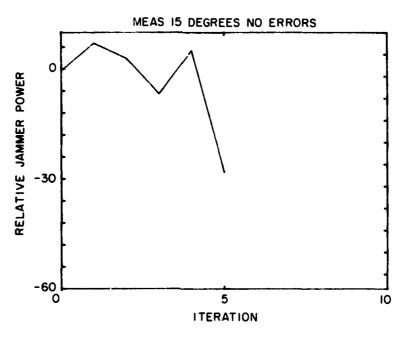


Figure 6. Relative Jammer Power vs Number of Iterations to Place a Null at  $\theta$  = 15°

In the second example, we imposed random phase errors with a 5° rms value on the quiescent element weights. Those errors result in the quiescent pattern in Figure 7. Also included in the figure is the cancellation beam along with its concomitant image beam formed to place a null at  $\theta$  = 12°. Figure 8 shows the perturbed and quiescent patterns for this example. Again there is little difference between the two patterns except near the directions of the cancellation beam and image beam. Here the -33.6-dB sidelobe at 12° was suppressed by 27.5 dB to produce a null 61.1 dB below the mainbeam. The behavior of the relative jammer power vs measurement curve of Figure 9 is similar to that for the phase error free example above, in that the 27.5 dB of sidelobe suppressions is obtained with the five measurements, including the initial reference plus the four measurements with  $\beta$  = 0°, 90°, 180°, and 270°.

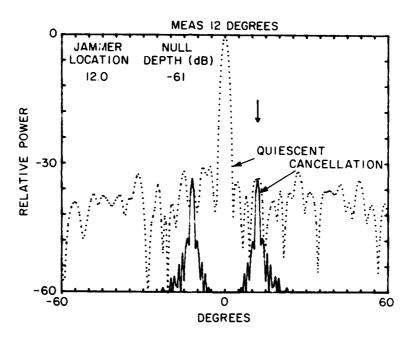


Figure 7. Quiescent Pattern Resulting From 5° rms Random Element Phase Errors and Cancellation Beam Formed to Place a Null at  $\theta$  = 12°

The next two examples are included to demonstrate the technique used when multiple nulls are desired. In the first example, each cancellation beam is a single sinc beam and in the second, each cancellation beam consists of a weighted sum of sinc beams to reduce the interaction between the cancellation beam as described in Section 2. For both examples the null directions are at the peaks of

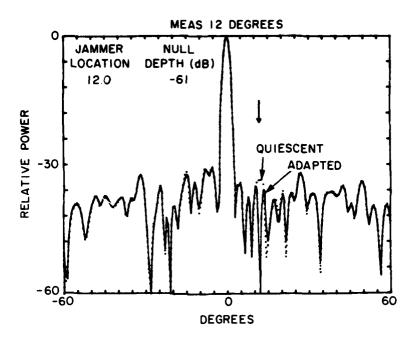


Figure 8. Adapted Pattern Compared to Quiescent Pattern for a Null at  $\theta$  = 12°

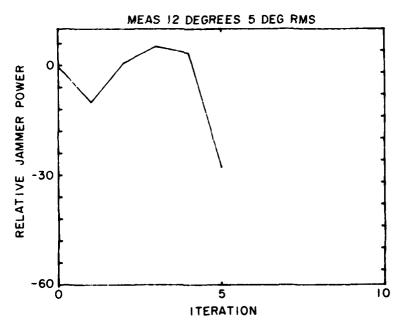


Figure 9. Relative Jammer Power vs Number of Iterations to Place a Null at  $\theta$  = 12°

the two adjacent sidelobes at  $\theta$  = 12.8° and  $\theta$  = 15° of the quiescent pattern shown in Figure 10.

The single sinc cancellation beams are shown in Figure 11. Notice the peak sidelobe of the beam at 12.8° occurs at about 15°, the peak of the second beam. Similarly, the peak sidelobe of the beam at 15° occurs near the peak of the first beam. Consequently, these two cancellation beams will interact. The perturbed pattern in Figure 12 shows that while both sidelobes have been reduced, only the one at  $\theta = 15^{\circ}$  has been reduced to the level achieved for a single null.

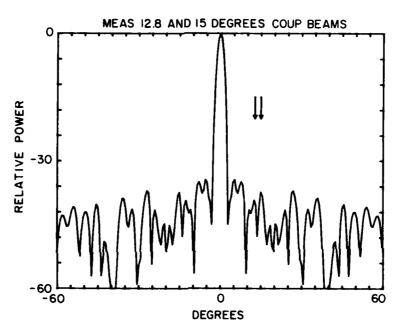


Figure 10. Quiescent Pattern for Multiple Null Example, Nulls at  $\theta$  = 12.8° and  $\theta$  = 15°

The relative jammer power vs measurement number for this case is given in Figure 13. The solid curve represe. the total interference, the dashed line indicates the contribution of the interference at 12.8°, and the dotted line indicates the contribution of the interference at 15.0°. The intensities of the two interference signals are equal, but the first is weighted by a -39-dB sidelobe and the second by a -37-dB sidelobe. Notice the power of the second interference fluctuates by a few dB while the measurements are taken with the second beam. Consequently, the residual power on each measurement is not a constant. This error in turn causes an error in the computed beam amplitudes and phases.

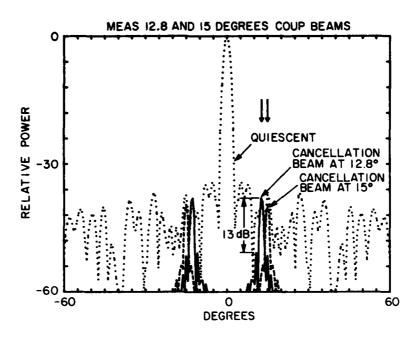


Figure 11. Individual Sinc Cancellation Beams to Null at  $\theta$  = 12.8° and  $\theta$  = 15°. Solid line shows beam formed at 12.8° and dashed line shows beam formed at 15°

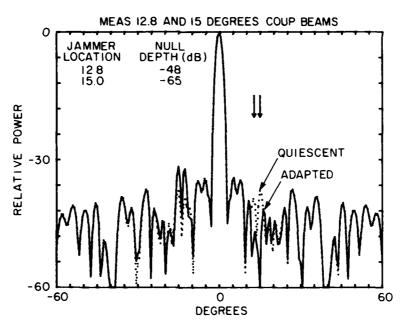


Figure 12. Adapted and Quiescent Patterns for Multiple Nulls Formed in  $\theta$  = 12.8° and  $\theta$  = 15° Directions With Individual Sinc Cancellation Beams

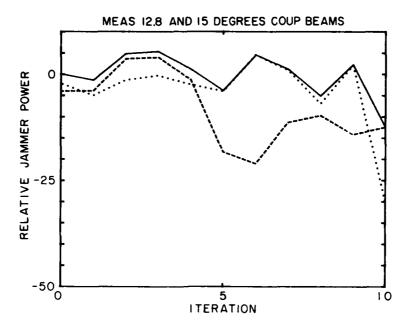


Figure 13. Relative Jammer Powers vs Iteration Number for Multiple Nulls Formed in  $\theta$  = 12.8° and  $\theta$  = 15° Directions With Individual Sinc Beams. Dashed line represents contribution of jammer at 12.8°, dotted line represents contribution of jammer at 15°, and solid line shows total jammer power

The interaction between the cancellation beams is reduced when the composite beams are used to form the nulls. Compare the cancellation beams in Figure 14 with those in Figure 11. While the beam at 12.8° in Figure 11 has a peak sidelobe near 15°, that in Figure 14 has a null. Similarly the beam at 15° in Figure 14 has a null at 12.8°, while that in Figure 11 has a peak. This reduction in beam interaction results in improved nulls as evidenced in Figure 15. As with previous examples there is minimal pattern distortion except near the null locations and their symmetric locations with respect to the mainbeam. The relative jammer power curves in Figure 16 further demonstrate the reduced interaction between the cancellation beams. Here we see the contribution of the interference at 15° is relatively constant during the measurements for the first cancellation beam. Likewise, the contribution of the interference at 12.8° varies little while the measurements are being taken for the second beam. Consequently, the residual power is reasonably constant during the measurement.

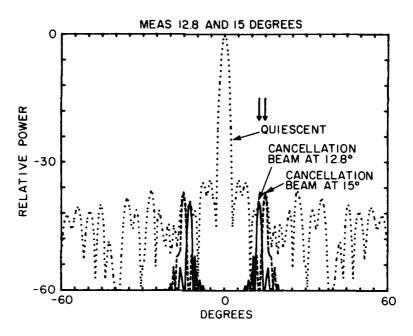


Figure 14. Decoupled Cancellation Beams for Nulls at  $\theta$  = 12.8° and  $\theta$  = 15°

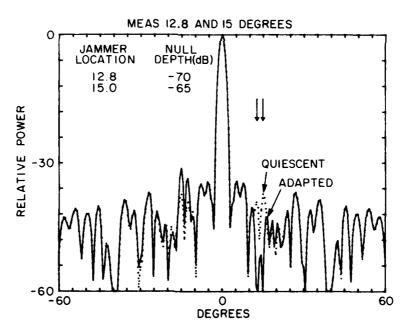


Figure 15. Adapted and Quiescent Patterns for Multiple Nulls Formed in  $\theta$  = 12.8° and  $\theta$  = 15° Directions With Decoupled Beams

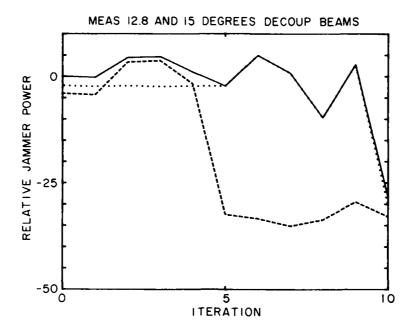


Figure 16. Relative Jammer Powers vs Iteration Number for Multiple Nulls Formed in  $\theta$  = 12.8° and  $\theta$  = 15° Directions With Decoupled Beams. Dashed line represents contribution of jammer at 12.8°, dotted line represents contribution of jammer at 15°, and solid line shows total jammer power

#### 5. CONCLUSIONS

The complex coefficients of analytic cancellation beams can be determined accurately by measuring the received power for four quadrature beam phase coefficients. This simple technique provides quality, albeit suboptimal, nulls with just five power measurements per null. The accuracy to which the residual power remains constant during the four power measurement determines the null depths that can be achieved. The effects of multipath and other phenomena not included in the simulation model will be evaluated with the experimental antenna.

This technique can also be used in conjunction with other more elaborate but slower converging techniques to obtain deeper nulls, where the measurement technique is used to obtain quick initial estimates for the cancellation beam coefficients, while the second technique is used to make fine adjustments to these coefficients.

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